

# Computergrafik

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Herbst 2016

# Today

## More shading

- Environment maps
- Reflection mapping
- Irradiance environment maps
- Ambient occlusion
- Reflection and refraction
- Toon shading

# More realistic illumination

- In real world, at each point in scene light arrives from all directions
  - Not just from point light sources
- Environment maps
  - Store “omni-directional” illumination as images
  - Each pixel corresponds to light from a certain direction

# Capturing environment maps

- “360 degrees” panoramic image
- Instead of 360 degrees panoramic image, take picture of mirror ball (light probe)



Light probes

[Paul Debevec, <http://www.debevec.org/Probes/>]

# Environment maps as light sources

## Simplifying assumption

- Assume light captured by environment map is emitted infinitely far away
- Environment map consists of directional light sources
  - Value of environment map is defined for each **direction**, independent of position in scene
- Use single environment map as light source at **all locations** in the scene
- Approximation!

# Environment maps as light sources

- How do you compute shading of a diffuse surface using an environment map?
- What is more expensive to compute, shading a diffuse or a specular surface?

# Environment maps applications

- Use environment map as “light source”



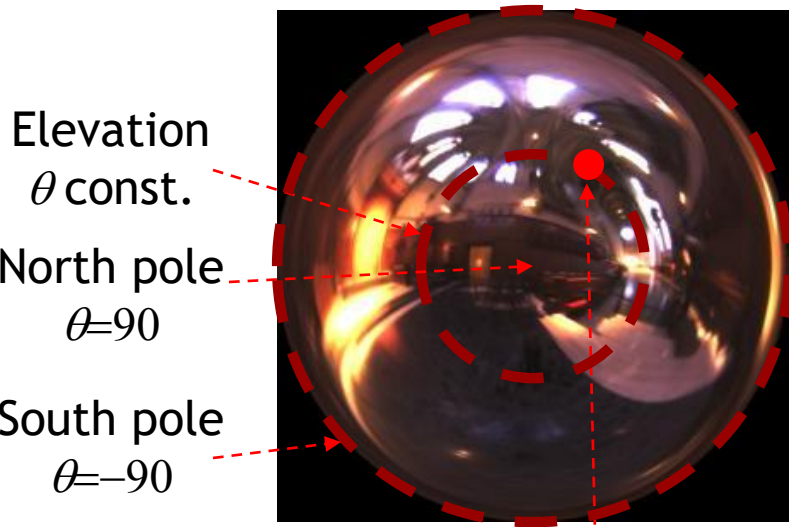
Global illumination  
[Sloan et al.]



Reflection mapping

# Sphere & cube maps

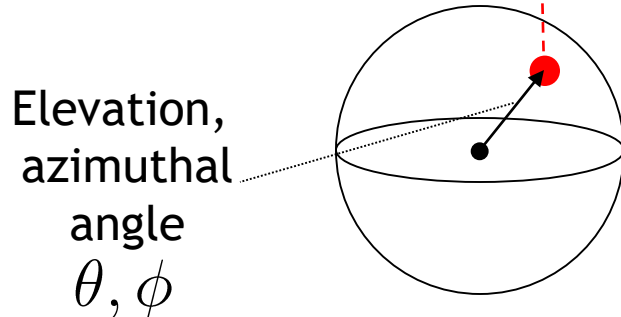
- Store incident light on sphere or on six faces of a cube



Elevation  
 $\theta$  const.

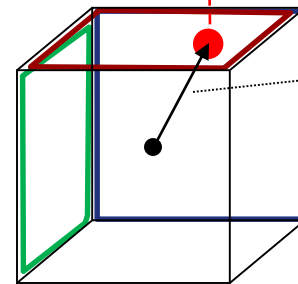
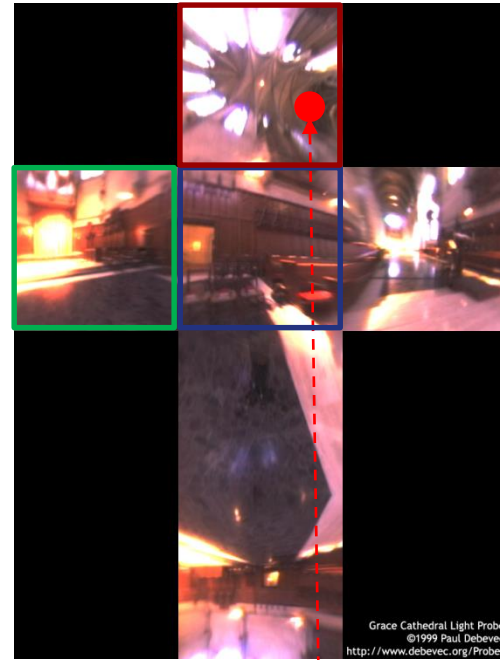
North pole  
 $\theta=90$

South pole  
 $\theta=-90$



Elevation,  
azimuthal  
angle  
 $\theta, \phi$

Spherical map



$x, y, z$

Cube map



# Cube maps in OpenGL

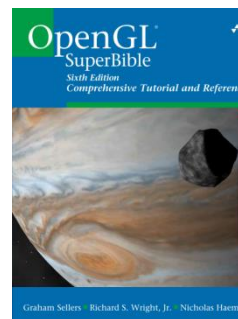
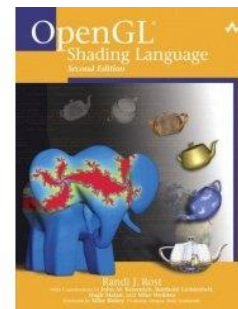
## Application setup

- Load, bind a cube environment map

```
glBindTexture(GL_TEXTURE_CUBE_MAP, ...);  
// the six cube faces  
glTexImage2D(GL_TEXTURE_CUBE_MAP_POSITIVE_X, ...);  
glTexImage2D(GL_TEXTURE_CUBE_MAP_NEGATIVE_X, ...);  
glTexImage2D(GL_TEXTURE_CUBE_MAP_POSITIVE_Y, ...);  
...  
glEnable(GL_TEXTURE_CUBE_MAP);
```

- More details

- “OpenGL Shading Language”, Randi Rost
- “OpenGL Superbible”, Sellers et al.
- Online tutorials



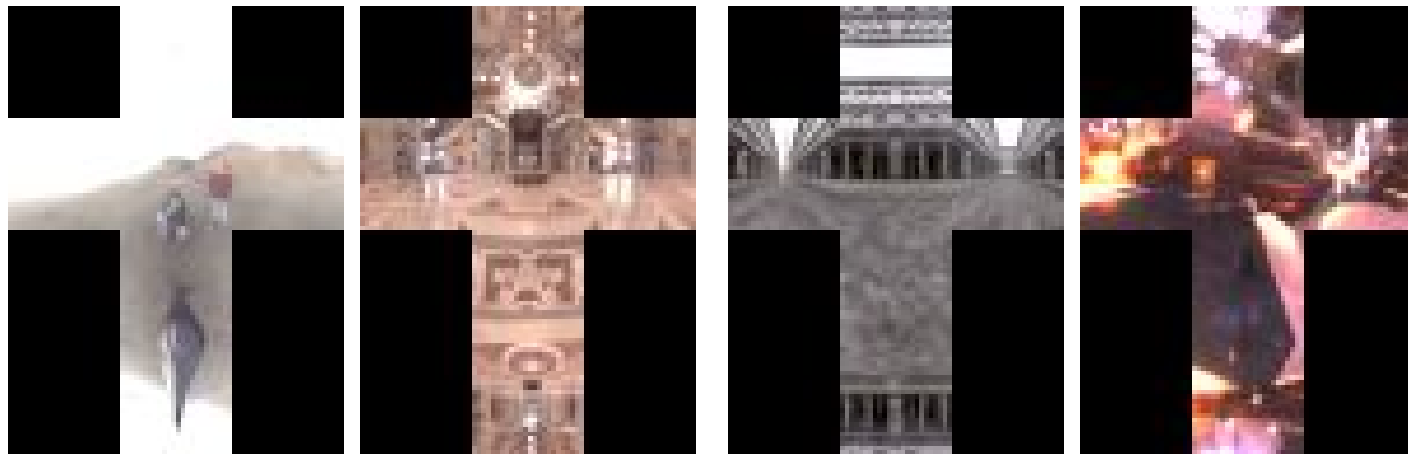
# Cube maps in OpenGL

## Look-up

- Given direction  $(x,y,z)$
- Largest coordinate component determines cube map face
- Dividing by magnitude of largest component yields coordinates within face
- Look-up function built into GLSL
  - Use  $(x,y,z)$  direction as texture coordinates to `samplerCube`

# Environment map data

- Also called „light probes“  
<http://www.debevec.org/Probes/>
- Tool for high dynamic range data (HDR)  
<http://projects.ict.usc.edu/graphics/HDRShop/>
- Pre-rendered light probes for games  
<http://docs.unity3d.com/Manual/LightProbes.html>



Light probes (<http://www.debevec.org/Probes/>)

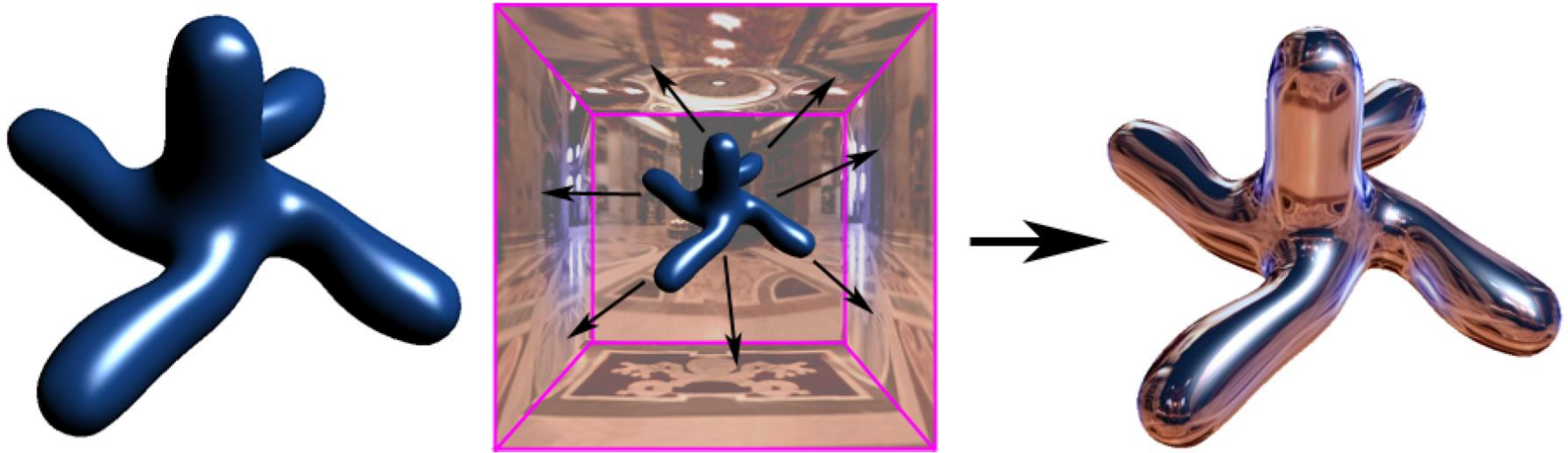
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# Reflection mapping

- Simulate mirror reflection
- Compute reflection vector at each pixel using view direction and surface normal
- Use reflection vector to look up cube map
- Rendering cube map itself is optional



Reflection mapping

# Reflection mapping in GLSL

## Vertex shader

- Compute viewing direction for each vertex
- Reflection direction
  - Use GLSL built-in `reflect` function
- Pass reflection direction to fragment shader

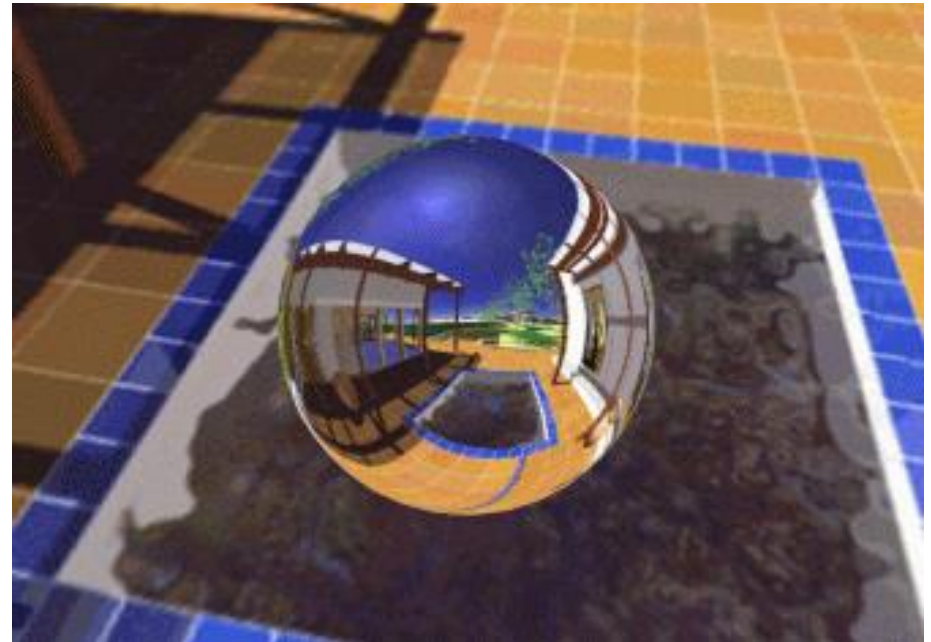
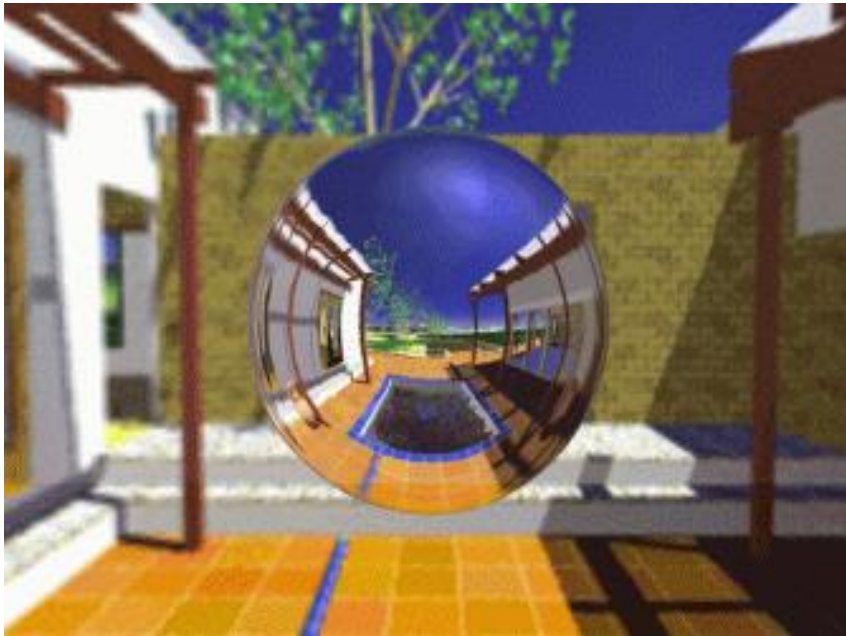
## Fragment shader

- Look-up cube map using interpolated reflection direction

```
in float3 refl;  
uniform samplerCube envMap;  
texture(envMap, refl);
```

# Reflection mapping examples

- Approximation, reflections are not accurate



[NVidia]

# Today

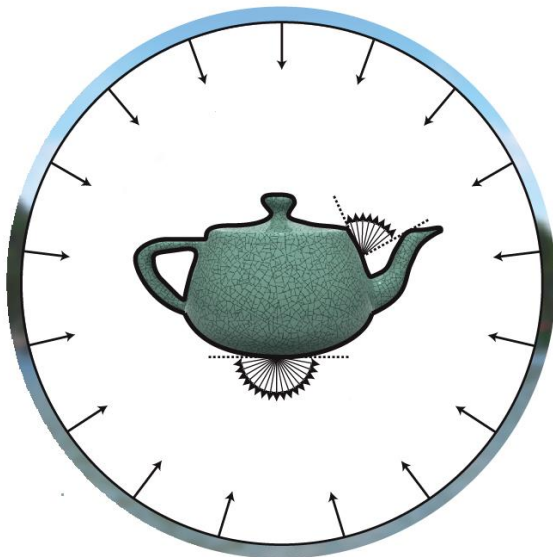
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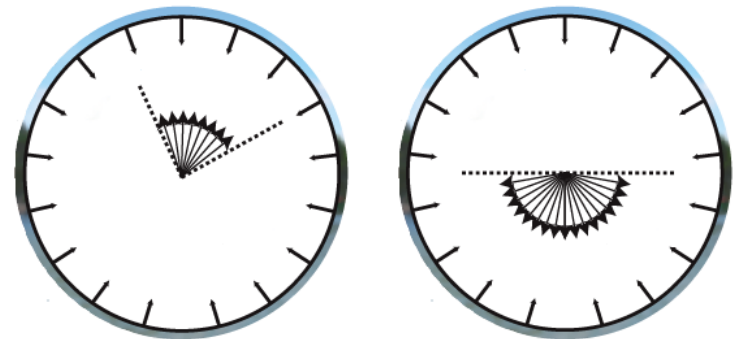


# Shading using environment map

- Assumption: distant lighting
  - Incident light is a **function of direction**, but not position
- Realistic shading requires
  - Take into account light from all directions
  - Include **occlusion**



Illumination from environment



Same environment map for both points  
“Illumination is a function of direction,  
but not position”

# Mathematical model

- Assume Lambertian (diffuse) material, BRDF  $k_d$ 
  - Ignore occlusion for now
- Illumination from point light sources

$$c = k_d \sum_i c_{l_i} (\mathbf{L}_i \cdot \mathbf{n})$$

- Illumination from environment map using **hemispherical integral**

$$c = k_d \int_{\Omega} c(\omega) (\omega \cdot \mathbf{n}) d\omega$$

- Directions  $\omega$
- Hemisphere of directions  $\Omega$
- Environment map, radiance from each direction  $c(\omega)$

# Irradiance environment maps

- Precompute **irradiance** as a function of normal

$$E(\mathbf{n}) = \int_{\Omega} c(\omega)(\omega \cdot \mathbf{n})d\omega$$

- Store as irradiance environment map
- Shading computation at render time
  - Depends only on normal, not position

$$c = k_d E(\mathbf{n})$$

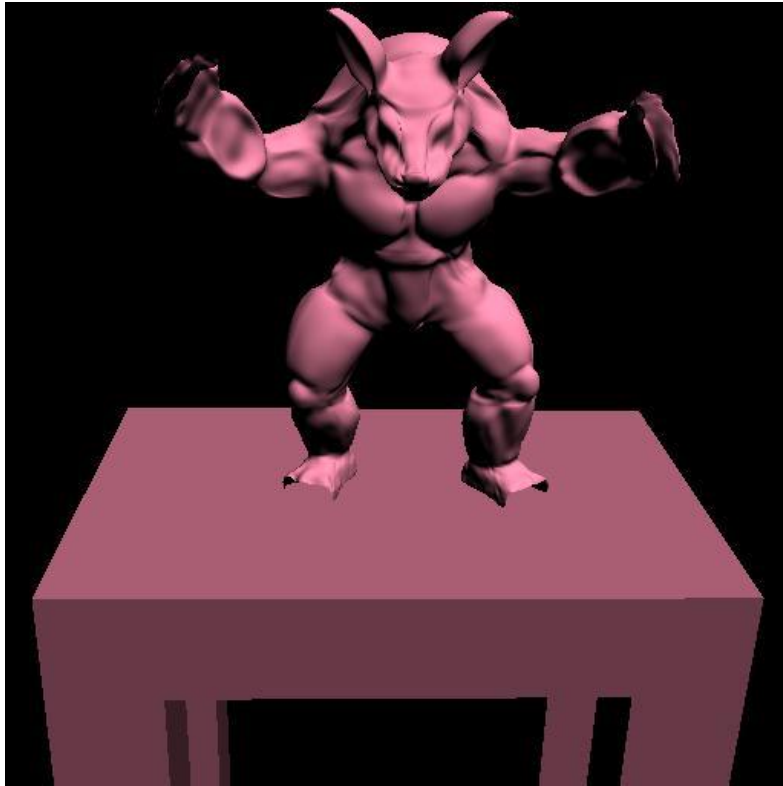


Environment map



Irradiance map

# Irradiance environment maps



Directional light



Environment illumination

Images from <http://www.cs.berkeley.edu/~ravir/papers/envmap/>

# Implementation

- Precompute irradiance map from environment
  - HDRShop tool, “diffuse convolution”  
<http://projects.ict.usc.edu/graphics/HDRShop/>
- At render time, look up irradiance map using surface normal
  - When object rotates, **rotate normal** accordingly
- Can also approximate glossy reflection
  - Blur environment map less heavily
  - Look up blurred environment map using reflection vector

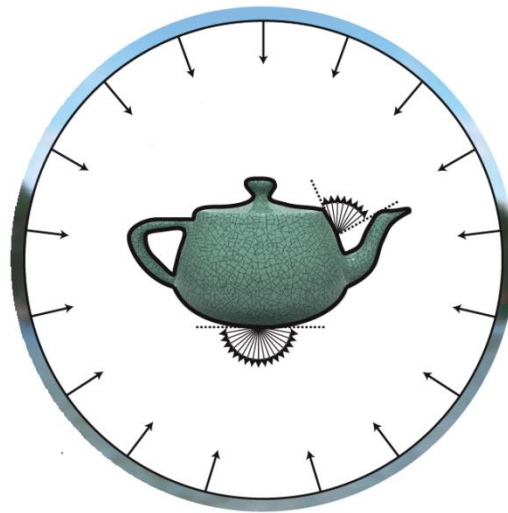
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# Including occlusion

- At each point, environment is partially occluded by geometry
- Add light only from un-occluded directions

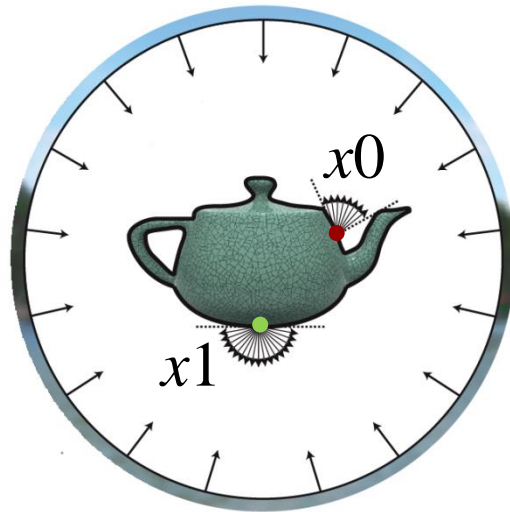


Visualization of un-occluded directions

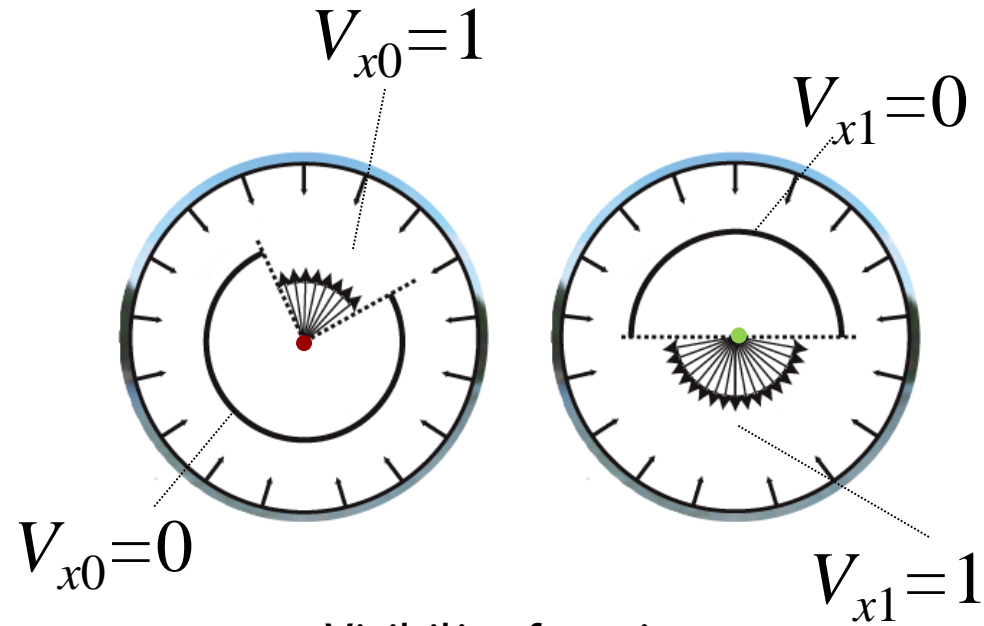
# Including occlusion

## Visibility function $V_x(\omega)$

- Binary function of direction  $\omega$
- Indicates if environment is occluded
- Depends on position  $x$



Environment map



Visibility functions

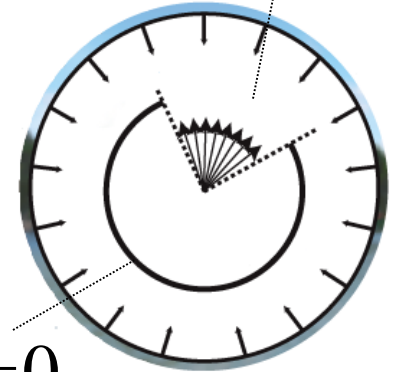


# Mathematical model

$V_x=1$

- Diffuse illumination with visibility

$$c = k_d \int_{\Omega} V_x(\omega) c(\omega) (\omega \cdot \mathbf{n}) d\omega$$



- Ambient occlusion

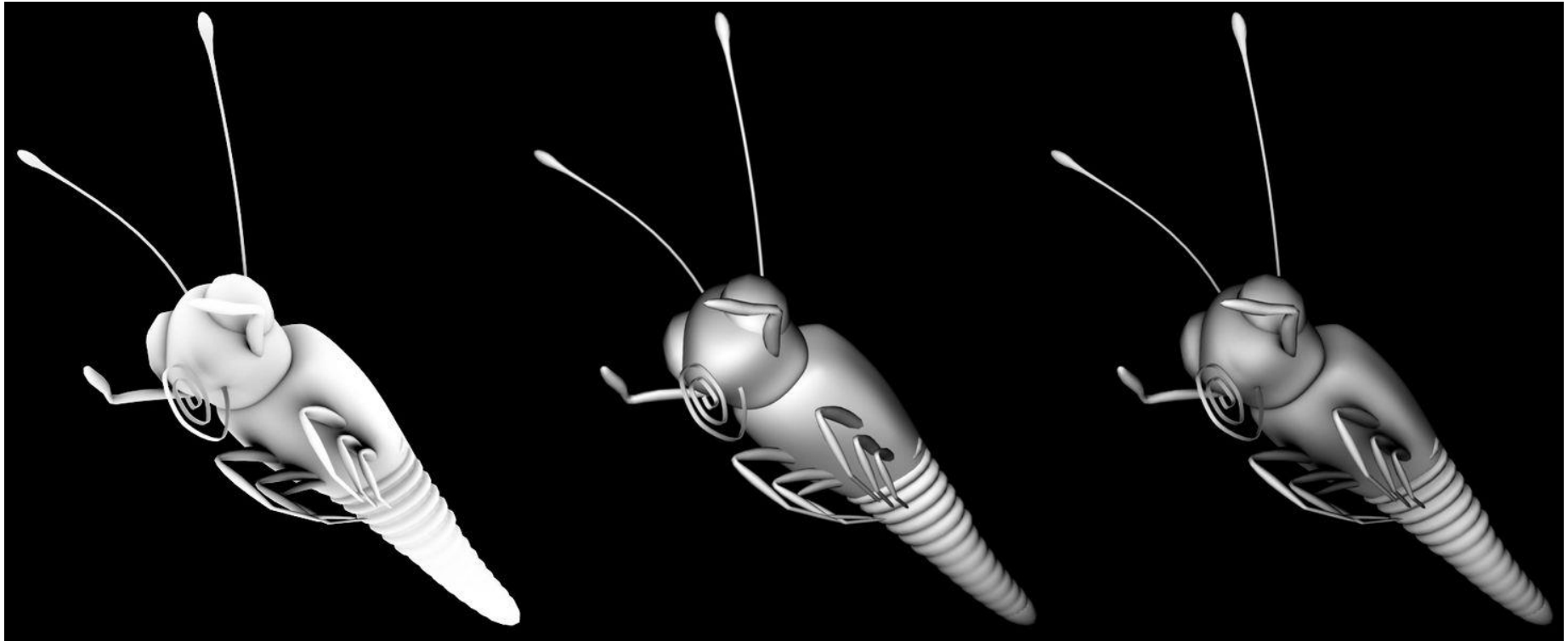
- “Fraction” of environment that is not occluded from a point  $x$

- Scalar value  $a_x = \int_{\Omega} V_x(\omega) (\omega \cdot \mathbf{n}) d\omega$

- **Approximation:** diffuse shading given by irradiance weighted by ambient occlusion

$$c = k_d a_x E(\mathbf{n}) \quad E(\mathbf{n}) = \int_{\Omega} c(\omega) (\omega \cdot \mathbf{n}) d\omega$$

# Ambient occlusion



Ambient occlusion

Diffuse shading

Ambient occlusion combined  
(using multiplication) with  
diffuse shading

# Implementation

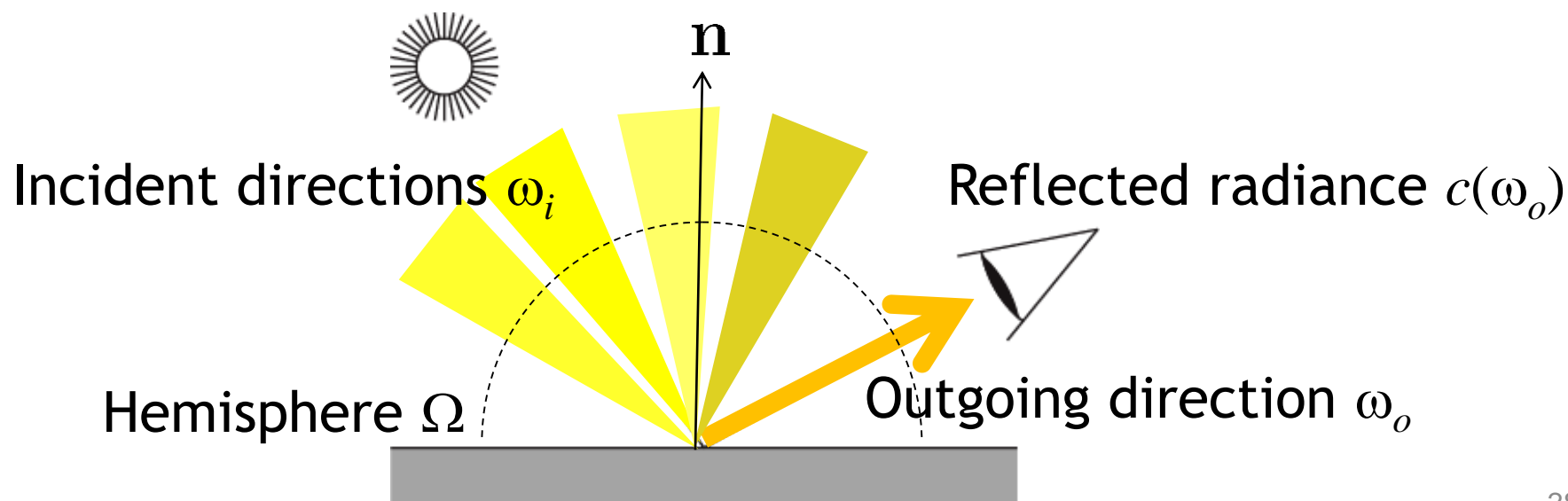
- **Precomputation** (off-line, before rendering)
  - Compute ambient occlusion on a per-vertex basis
  - Using ray tracing
  - Free tool that saves meshes with per-vertex ambient occlusion  
<http://www.xnormal.net/>
- **Caution**
  - Basic pre-computed ambient occlusion does not work for animated objects

# Shading integral

- Ambient occlusion with irradiance environment maps is crude approximation to general **shading integral**

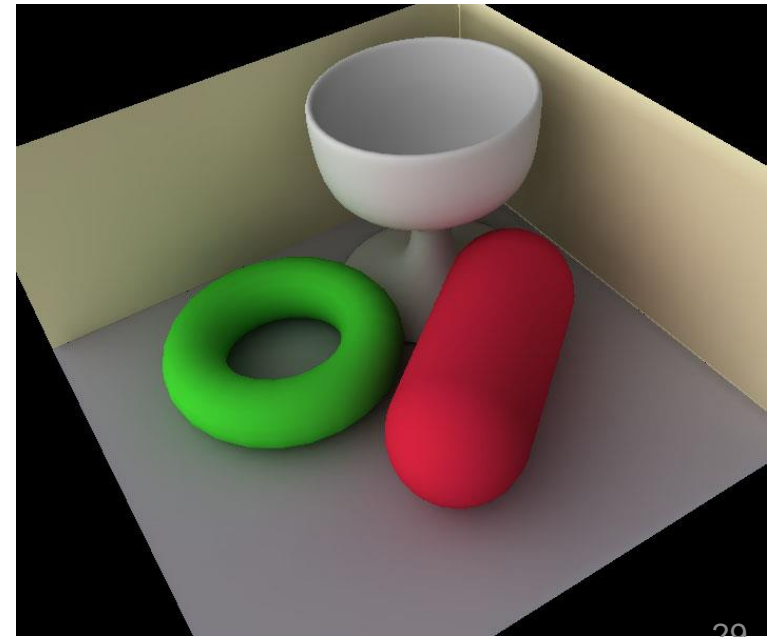
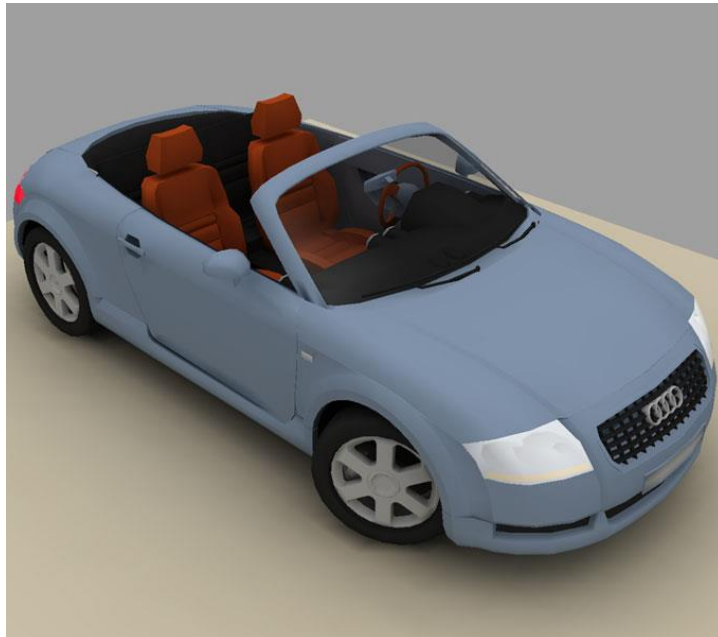
$$c(\omega_o) = \int_{\Omega} V_x(\omega_i) c(\omega_i) f(\omega_o, \omega_i) (\omega_i \cdot \mathbf{n}) d\omega_i$$

- BRDF for (non-diffuse) material  $f(\omega_o, \omega_i)$



# Shading integral

- Accurate evaluation is expensive to compute
  - Requires numerical integration
- Many tricks for more accurate and general approximation than ambient occlusion and irradiance environment maps exist
  - Spherical harmonics shading  
<http://www.research.scea.com/gdc2003/spherical-harmonic-lighting.pdf>



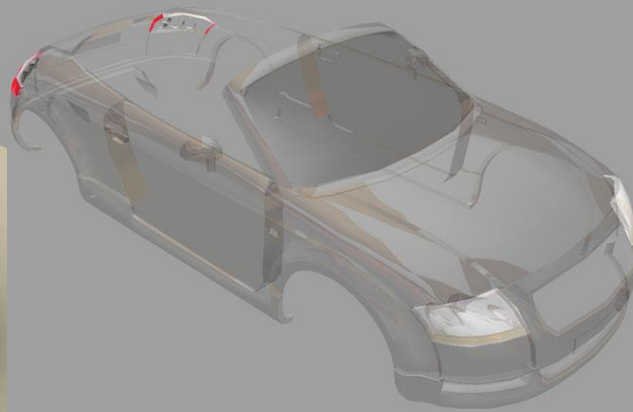
# Note

- Visually interesting results using combination (sum) of diffuse shading with ambient occlusion and reflection mapping

Diffuse shading with ambient occlusion



Reflection mapping



Combination (sum)



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# Reflection & refraction

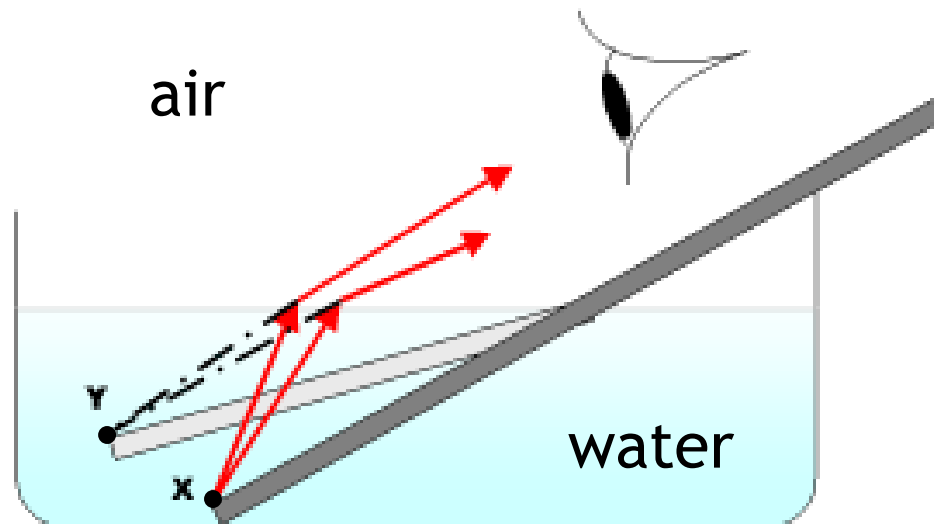




# Refraction

<http://en.wikipedia.org/wiki/Refraction>

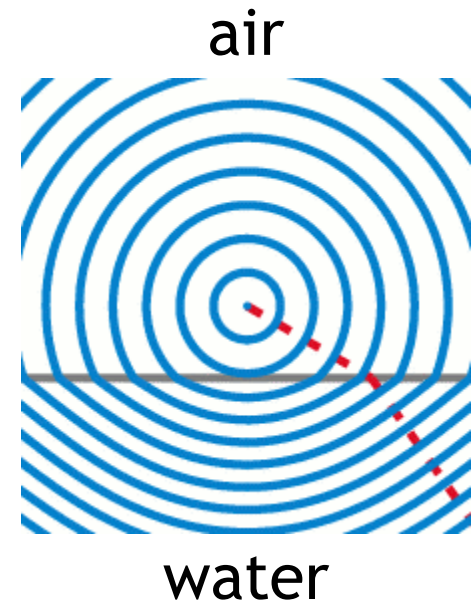
- Light rays that travel from one medium to another are bent
- To the viewer, object at location  $x$  appears to be at location  $y$



# Index of refraction

[http://en.wikipedia.org/wiki/Refractive\\_index](http://en.wikipedia.org/wiki/Refractive_index)

- Speed of light depends on medium
  - Speed of light in vacuum  $c$
  - Speed of light in medium  $v$
- Index of refraction  $n=c/v$ 
  - Air 1.00029
  - Water 1.33
  - Acrylic glass 1.49
- “Change in phase velocity leads to bending of light rays”



# Snell's law

[http://en.wikipedia.org/wiki/Snell's\\_law](http://en.wikipedia.org/wiki/Snell's_law)

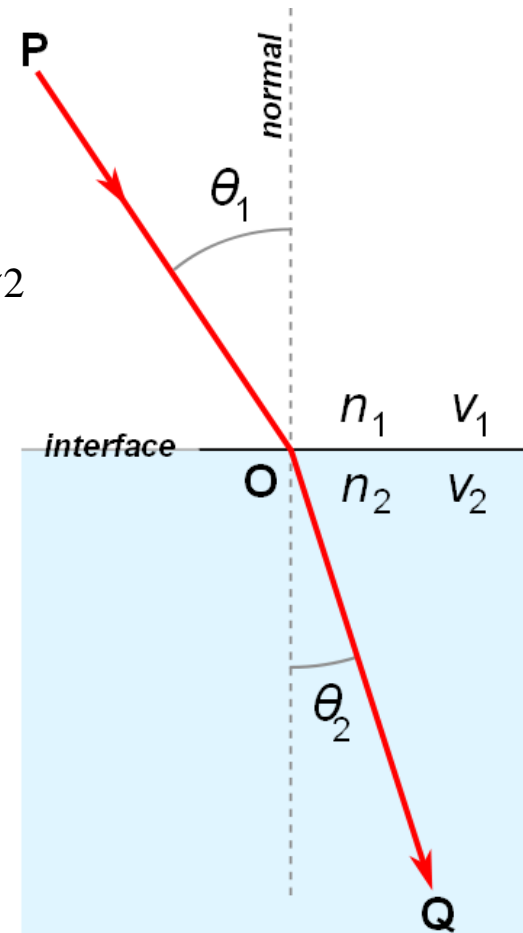
- Ratio of sines of angle of incidence  $\theta_1$  and refraction  $\theta_2$  is equal to opposite ratio of indices of refraction  $n_1, n_2$

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1}$$

- Vector form in 3D

$$\mathbf{r} = \frac{n_1}{n_2} \mathbf{v} + \left( \frac{n_1}{n_2} \cos \theta_1 + \cos \theta_2 \right) \mathbf{n}$$

- Viewing, refracted direction  $\mathbf{v}, \mathbf{r}$
- Normal vector  $\mathbf{n}$



# Total internal reflection

[http://en.wikipedia.org/wiki/Total\\_internal\\_reflection](http://en.wikipedia.org/wiki/Total_internal_reflection)

- Angle of refracted ray

$$\theta_2 = \arcsin \left( \theta_1 \frac{n_1}{n_2} \right)$$

- Critical angle

$$\theta_c = \frac{n_2}{n_1}$$

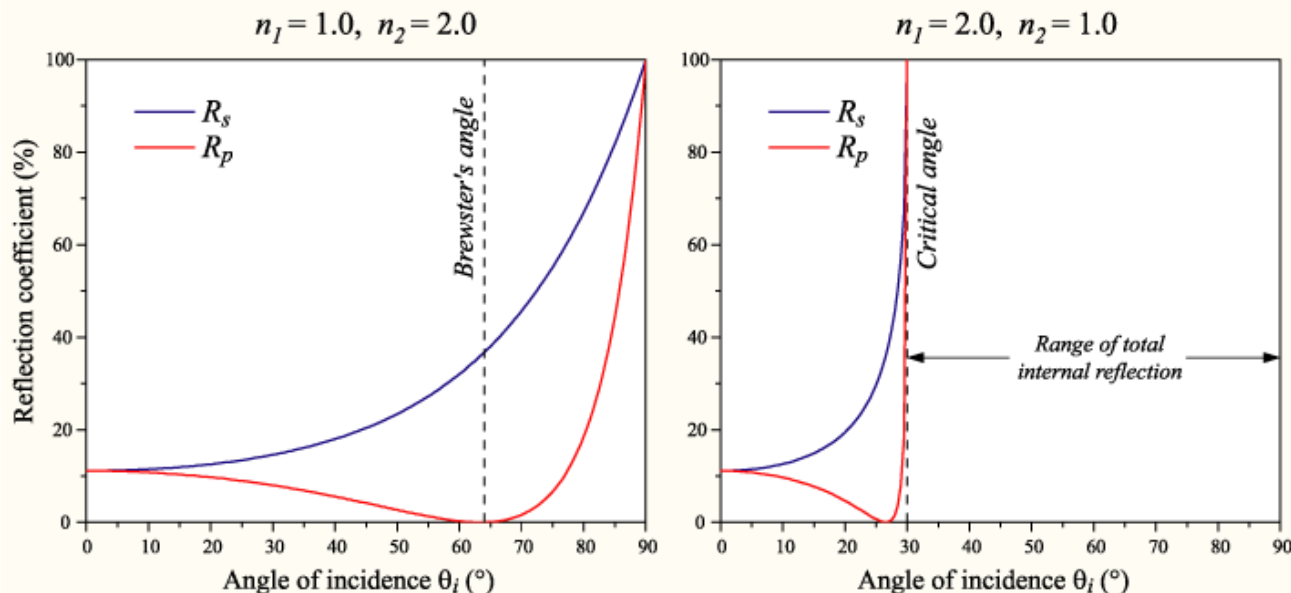
- If  $\theta_1 = \theta_c$  we get  $\theta_2 = \pi/2$ , refracted ray is parallel to interface
- If  $\theta_1 > \theta_c$  we have total internal reflection (light ray does not cross interface between media)



# Fresnel equations

[http://en.wikipedia.org/wiki/Fresnel\\_equations](http://en.wikipedia.org/wiki/Fresnel_equations)

- When light travels from one medium to another, both reflection and refraction may occur
- Fresnel equations describe fraction of intensity of light that is reflected and refracted
  - Depend on polarization of light ( $R_s$ ,  $R_p$  in plots)



# Fresnel equations

- Fresnel equations are relatively complex to evaluate
- In graphics, often use Schlick's approximation
  - Ratio  $F$  between reflected and refracted light
  - Indices of refraction  $n_1, n_2$

[https://en.wikipedia.org/wiki/Schlick%27s\\_approximation](https://en.wikipedia.org/wiki/Schlick%27s_approximation)

$$F = f + (1 - f)(1 - \mathbf{v} \cdot \mathbf{n})^5 \qquad f = \frac{\left(1.0 - \frac{n_1}{n_2}\right)^2}{\left(1.0 + \frac{n_1}{n_2}\right)^2}$$

# Implementation

- Accurate implementation requires ray tracing
- For interactive graphics, approximation using environment maps
  - Use **reflected and refracted** rays to look up **environment map**
  - Use Fresnel equations to determine fraction of reflected and refracted light
  - Does not take into account geometry after first bounce (i.e., surface intersection)
  - Assumes illumination is infinitely far away

# Vertex shader

```
const float Eta = 0.67;           // Ratio of indices of refraction (air -> glass)
const float FresnelPower = 10.0; // Controls degree of reflectivity at grazing angles

const float F = ((1.0 - Eta) * (1.0 - Eta)) / ((1.0 + Eta) * (1.0 + Eta));

varying vec3  Reflect;
varying vec3  Refract;
varying float Ratio;

void main(void)
{
    vec4 ecPosition = gl_ModelViewMatrix * gl_Vertex;
    vec3 ecPosition3 = ecPosition.xyz / ecPosition.w;

    vec3 i = normalize(ecPosition3);
    vec3 n = normalize(gl_NormalMatrix * gl_Normal);

    Ratio = F + (1.0 - F) * pow((1.0 - dot(-i, n)), FresnelPower);

    Refract = refract(i, n, Eta);
    Refract = vec3(gl_TextureMatrix[0] * vec4(Refract, 1.0));

    Reflect = reflect(i, n);
    Reflect = vec3(gl_TextureMatrix[0] * vec4(Reflect, 1.0));

    gl_Position = ftransform();
}
```

**CAUTION: need to update  
this for OpenGL 3 compliance**



# Fragment shader

- Application needs to set up cube map

```
varying vec3  Reflect;
varying vec3  Refract;
varying float Ratio;

uniform samplerCube cubemap;

void main(void)
{
    vec3 refractColor = vec3 (textureCube(cubemap, Refract));
    vec3 reflectColor = vec3 (textureCube(cubemap, Reflect));

    vec3 color      = mix(refractColor, reflectColor, Ratio);

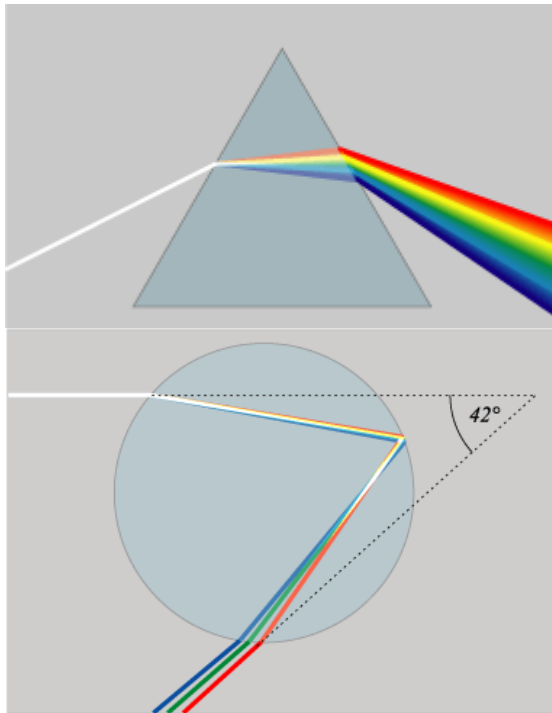
    gl_FragColor = vec4(color, 1.0);
}
```

**CAUTION: need to update  
this for OpenGL 3 compliance**

# Chromatic dispersion

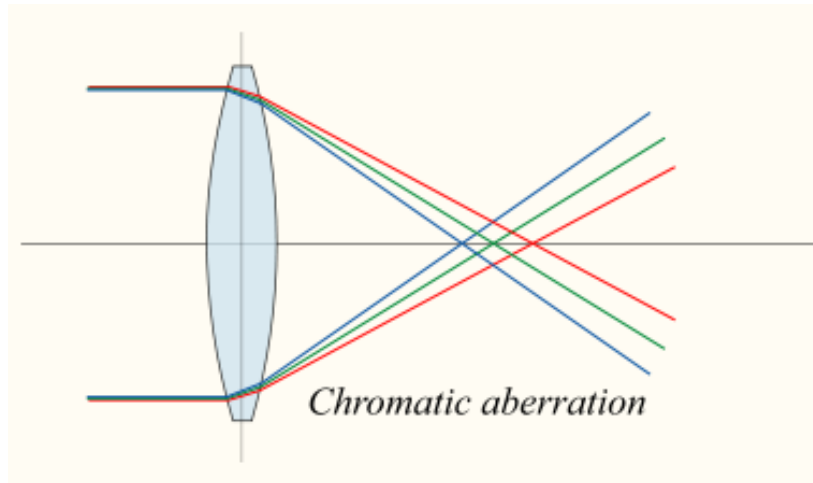
[http://en.wikipedia.org/wiki/Dispersion\\_\(optics\)](http://en.wikipedia.org/wiki/Dispersion_(optics))

- Phase velocity (i.e., index of refraction) in many media depends on wavelength/frequency
  - Dispersive media
- Different colors refract at different angles



# Chromatic dispersion

- In the context of camera lenses, chromatic aberration
  - Try to use *achromatic* lenses



# Implementation

- Approximate dispersion by using three different ratios of indices of refraction for R,G,B channels
  - Glass: 0.65, 0.67, 0.69
- Perform separate look-ups for R,G,B channels in environment map

# Today

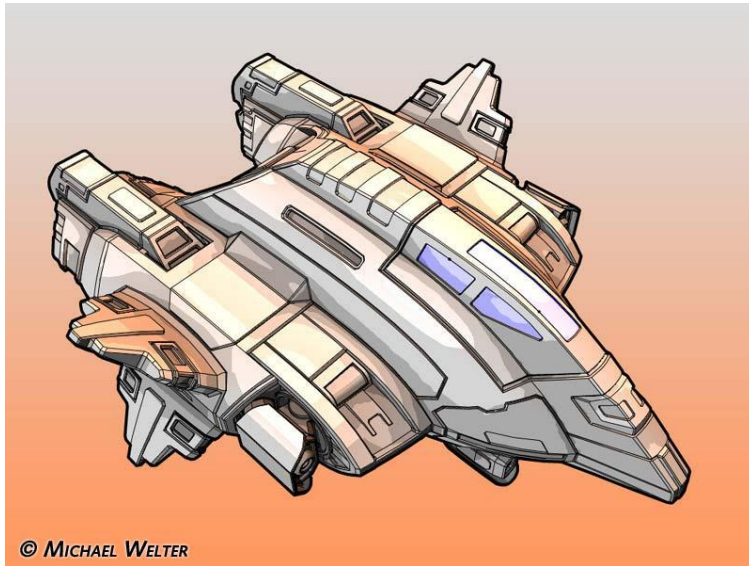
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- **Toon shading**

# Toon shading

- Simple cartoon style shader
- Emphasize silhouettes
- Discrete steps for diffuse shading, highlights
- Sometimes called CEL shading

[http://en.wikipedia.org/wiki/Cel-shaded\\_animation](http://en.wikipedia.org/wiki/Cel-shaded_animation)



Off-line toon shader



GLSL toon shader

# Toon shading

- Silhouette edge detection

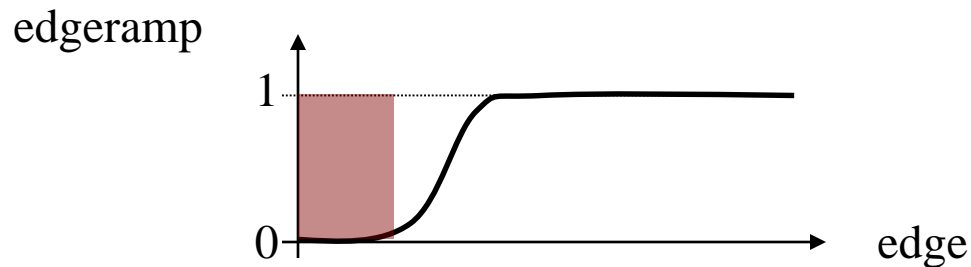
- Compute dot product of viewing direction  $\mathbf{v}$  and normal  $\mathbf{n}$

$$\text{edge} = \max(0, \mathbf{n} \cdot \mathbf{v})$$



- Use 1D texture to define edge ramp

```
uniform sampler1D edgeramp; e=texture1D(edgeramp,edge);
```



# Toon shading

- Compute diffuse and specular shading

$$\text{diffuse} = \mathbf{n} \cdot \mathbf{L} \quad \text{specular} = (\mathbf{n} \cdot \mathbf{h})^s$$

- Use 1D textures `diffuseramp`, `specularramp` to map diffuse and specular shading to colors

- Final color

```
uniform sampler1D diffuseramp;
```

```
uniform sampler1D specularramp;
```

```
c = e * (texture(diffuse,diffuseramp)+  
         texture(specular,specularramp));
```



# Tools

- Nvidia developer page

<http://developer.nvidia.com/page/home.html>

# Next time

- Bump mapping, shadows